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Creation and annihilation of traffic jams in a stochastic asymmetric exclusion model with open boundaries: a computer simulation

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Abstract. The creation and annihilation of traffic jams are studied by a computer simulation. The one-dimensional (1D) fully-asymmetric exclusion model with open boundaries for parallel update is extended to take into account stochastic transition of particles (cars) where a particle moves ahead with transition probability p_t if the forward nearest neighbour is not occupied. Near $p_t = 1$, the system is drived asymptotically into a steady state exhibiting a self-organized criticality. In the self-organized critical state, a traffic jam (start–stop wave) and an empty wave are created at the same time when a car stops temporarily. The traffic jam disappears by colliding with the empty wave. The coalescence process between traffic jams and empty waves is described by the ballistic annihilation process with pair creation. The resulting problem near $p_t = 1$ is consistent with the ballistic process in the context of 1D crystal growth studied by Krug and Spohn. The typical lifetime $\langle m \rangle$ of start–stop waves scales as $\langle m \rangle \approx \Delta p_t^{-0.54\pm0.04}$ where $\Delta p_t = 1 - p_t$. It is shown that the cumulative distribution $N_m(\Delta p_t)$ of lifetimes satisfies the scaling form $N_m(\Delta p_t) \approx \Delta p_t^{1.1} f(m\Delta p_t^{0.54})$. Also, the typical interval $\langle s \rangle$ between consecutive traffic jams scales as $\langle s \rangle \approx \Delta p_t^{-0.50\pm0.04}$. The cumulative interval distribution $N_s(\Delta p_t)$ of traffic jams satisfies the scaling form $N_s(\Delta p_t) \approx \Delta p_t^{0.50} g(s\Delta p_t^{0.50})$. For $p_t < 1$, no scaling holds.

1. Introduction

The 1D exclusion model is one of the simplest examples of a driven system. The model has been extensively investigated to understand systems of interacting particles [1–5]. The 1D exclusion model is used to study the microscopic structure of shocks [6, 7] and is closely linked to growth processes [8–11]. The 1D fully asymmetric simple-exclusion model can be formulated into traffic flow problems on a highway [12]. The 1D exclusion model with parallel update is consistent with the deterministic cellular automaton (CA) 184 in the classification by Wolfram [13]. The 2D versions of the asymmetric simple-exclusion model were applied to the traffic-jam problem in an entire city [14–16].

CA models are being applied successfully to simulations of traffic. Nagel and Schreckenberg [12] extended the 1D fully-asymmetric exclusion model to take car velocity into account in order to simulate freeway traffic. Nagatani [17] studied the clustering of traffic in the extended asymmetric exclusion model taking into account the difference between the inherent velocities of individual cars. Ben-Naim *et al* [18] analysed the kinetic clustering of cars in a simple aggregation model.

Recently, phenomena exhibiting self-organized criticality have attracted considerable attention [19, 20]. Bak *et al* [19] introduced the notion of self-organized criticality. They showed that the sandpile naturally evolves into a critical state through a self-organization

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process. This critical state is characterized by no intrinsic length or time scales. Nagel and Herrmann [21] showed that the open-boundary version of the 1D traffic flow exhibits a self-organized criticality, providing enough input and output of cars at the boundaries. Nagel and co-worker [22, 23] studied the lifetimes of simulated traffic jams for freeway traffic and found emergent traffic jams with a self-similar appearance. However, the CA model is not simple since it is described by the CA rule of seven states. The dependence of scaling behaviour upon the system size is unclear. Nagatani [24] showed that introducing injection or extraction into the 1D fully-asymmetric exclusion model with periodic boundary drives the system asymptotically into a steady state exhibiting a self-organized criticality. In the 1D model with periodic boundary, the injection or extraction of particles maintain the self-organized critical state. It was shown that the typical jam-interval $\langle s\rangle$ scales as

$$\langle s\rangle \approx L^{\nu} \qquad \text{with } \nu = 0.62 \pm 0.04 \tag{1}$$

and the jam-interval distribution $n_s(L)$ satisfies the finite-size scaling form

$$n_s(L) \approx L^{-\beta} f(s/L^{\nu}) \qquad \text{with } \beta = 2\nu \tag{2}$$

where L is the system size. The typical lifetime $\langle m\rangle$ of jams scales as

$$\langle m\rangle \approx L^{\nu}. \tag{3}$$

The lifetime distribution $n_m(L)$ satisfies the finite-size scaling form

$$n_m(L) \approx L^{-\gamma} g(m/L^{\nu}) \qquad \text{with } \gamma = 1 + \nu. \tag{4}$$

It was also found that adding a temporary stopping of particles to the 1D fully-asymmetric exclusion model with open boundaries drives the system asmptotically into a steady state exhibiting a self-organized criticality [25]. In the 1D model with open boundaries, the self-organized critical state is maintained without injection or extraction of particles. The typical interval $\langle s\rangle$ and lifetime $\langle m\rangle$ scale in the same form as equations (1) and (3) with a different value for the scaling exponent $\nu = 0.5$. The interval and lifetime distributions also satisfy the same finite-size scaling form as equations (2) and (4) with a different value for the scaling exponent $\nu = 0.5$. Comparison with the work of Nagel [22, 23] is difficult since the definition of jam size in [22, 23] is very different from that used here.

In this paper, I present the 1D stochastic fully-asymmetric exclusion model to take into account the stochastic transition of particles. A particle moves ahead with transition probability p_t if the forward nearest neighbour is unoccupied. The model describes the 1D traffic flow on a highway. Cars flow from the inlet and flow out at the exit on a highway where cars move or sometimes stop. When $p_t = 1$, the model is consistent with the 1D deterministic fully-asymmetric exclusion model. At $p_t = 1$, the traffic flow is driven into the maximal velocity phase in which cars are distributed with a particular configuration with alternate spacing. Then, if a car stops temporarily, a traffic jam and an empty wave are created. Near $p_t = 1$, the traffic jams and empty waves appear one after another. The traffic jam disappears by colliding with the empty wave. The creation and annihilation of traffic jams reaches a steady state. The process is described by the ballistic annihilation process [26, 27] with pair creation. The resulting problem near $p_t = 1$ agrees with that in the context of one-dimensional crystal growth which was studied by Krug and Spohn [28]. We study the scaling behaviour of the interval and lifetime. We show that the self-organized criticality occurs near $p_t = 1$ without introducing injection or temporary stopping. We find that the interval and lifetime distributions satisfy scaling forms different from equations (2) and (4). However, the scaling does not hold for $p_t < 1$.

The organization of the paper is as follows. In section 2 we present the model and the simulation method. In section 3 we give the simulation result. In section 4 we compare our result with the previous results obtained from the different models. Section 5 presents the summary.

2. Model and simulation

We consider the traffic flow of cars in which cars flow from the inlet and flow out at the exit on a highway with fluctuating velocity. Car velocity fluctuates due to irregular road conditions, the whim of drivers or interference with other cars. Cars flow smoothly without fluctuation of car velocity. The fluctuating velocity induces an irregular traffic flow into a smooth flow of cars. We study the irregular traffic flow. We present a 1D stochastic CA model of traffic flow on a highway. We extend the 1D deterministic fully-asymmetric simple-exclusion model with open boundaries for parallel update to take fluctuating velocity into account. Its fluctuating velocity is taken into account as the stochastic transition of particles in the 1D fully-asymmetric exclusion model. The 1D fully-asymmetric exclusion model, which describes a system of particles hopping in a preferred direction with a hard core interaction, was solved exactly in the case of open boundaries [1]. In the model, a particle is added at the inlet site with probability α if the inlet site is empty and a particle is removed from the exit site with probability β if this site is occupied. It is known that the maximal current ($J = \frac{1}{4}$) phase appears if $\alpha > \frac{1}{2}$ and $\beta > \frac{1}{2}$.

We consider the case of $\alpha = \beta = 1$. In the 1D deterministic simple-exclusion model with open boundaries for parallel update, the maximal current is given by $\frac{1}{2}$. In the maximal current phase, particles move with the maximal velocity 1, particles are distributed in the configuration with alternate spacing and the density of particles is $\frac{1}{2}$. In our stochastic model, particles move ahead with probability p_t if the forward nearest neighbour is empty, or otherwise, particles stop with probability $1 - p_t$. Particles move or stop stochastically. In the limit of $p_t = 1$, our model reduces to the 1D deterministic simple-exclusion model. Near $p_t = 1$, particles move close to the maximal current phase. Some particles stop temporarily. Then, a traffic jam (start–stop wave) and an empty wave are created at the same time. The start–stop wave consists of two connected particles and propagates backward. The empty wave consists of two connected empty sites and propagates forward. When the start–stop waves collide with the empty waves, they disappear. The start–stop and empty waves appear one after another. Thus, introducing stochastic transition into the deterministic asymmetric exclusion model drives the system into a steady state. We study the scaling behaviour of the creation and disappearance of the start–stop waves in the steady state.

Our stochastic CA model is defined on a 1D lattice of L sites with open boundaries. Each site is either empty or occupied by one particle (or one car). For an arbitrary configuration, one update of the system is performed in parallel for all cars. Particles move ahead by one step with probability p_t unless the forward nearest-neighbour site is occupied by another particle, otherwise they stop with probability $1 - p_t$. If particles are blocked ahead by another particle, they do not move even if the blocking particle moves out of the site during the same time step. A particle is added at the inlet site 1 if the inlet site is empty. If the exit site L is occupied by one particle, its particle is removed from the exit site.

3. Simulation result

We have performed simulations of the stochastic CA model starting with an ensemble of random initial conditions where the system size is $L = 10^3$–10^5 and the initial density of particles is $p_0 = 0.0$–1.0. Each run is calculated up to 10^4–10^5 time steps. For illustration,

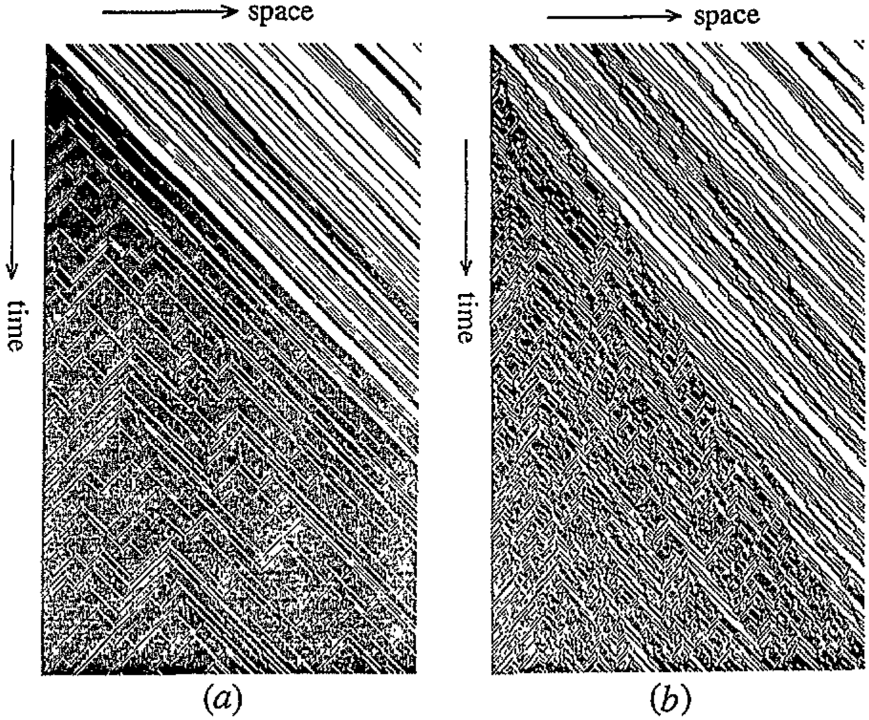

Figure 1. The typical patterns of particles up to 500 time steps where the system size is $L = 300$. The horizontal and vertical directions indicate space and time respectively. A particle is indicated by a dot. The trajectory of a particle is represented by a curve. The grey region is the area in which particles move with the interval of two sites. The black region represents the start–stop wave in which particles are stopped because their progress is blocked by another particle. (a) The pattern for the transition probability $p_t = 0.99$. The white region represents the empty wave. The start–stop and empty waves are created at the same time. The start–stop wave disappears by colliding with the empty wave. (b) The pattern for the transition probability $p_t = 0.9$.

figure 1 shows the typical patterns for the initial density $p_0 = 0.2$ up to 500 time steps where the system size is $L = 300$. Patterns (a) and (b) indicate, respectively, those for the transition probability $p_t = 0.99$ and 0.9. The horizontal direction indicates the direction in which particles move ahead. The vertical direction indicates time. A particle is indicated by a dot. The trajectory of a particle is represented by a curve. In pattern (a), the grey region indicates that in which particles move with the interval of two sites. The local density of particles is $p = 0.5$. Particles within the region move with the maximal velocity 1. The black region represents the appearance of a start–stop (or traffic jam) in which particles are stopped because their progress is blocked by another particle. The start–stop wave propagates backward and consists of two connected particles. The white region indicates the appearance of an empty wave which consists of two connected empty sites. Its empty wave propagates forward. The start–stop wave disappears by colliding with the empty wave. The coalescence process between the start–stop and empty waves is similar to a ballistic two-species annihilation reaction, $A + B \to \Phi$. The start–stop wave (or traffic jam) disappears with various lifetimes. In the case of $\Delta p_t (= 1 - p_t) \ll 1$ (pattern (a)), the mean density approaches the value $\frac{1}{2}$ of a steady state. The start–stop wave appears clearly. Therefore, it is expected that the scaling phenomenon appears near $p_t = 1$. However, in the case (pattern (b)) in which the condition $\Delta p_t \ll 1$ is not satisfied, the density fluctuates largely and the coalescence process between the start–stop and empty waves becomes obscure. The process in pattern (b) cannot be described by the ballistic annihilation process. Therefore, the scaling breaks down when the condition $\Delta p_t \ll 1$ is not satisfied.

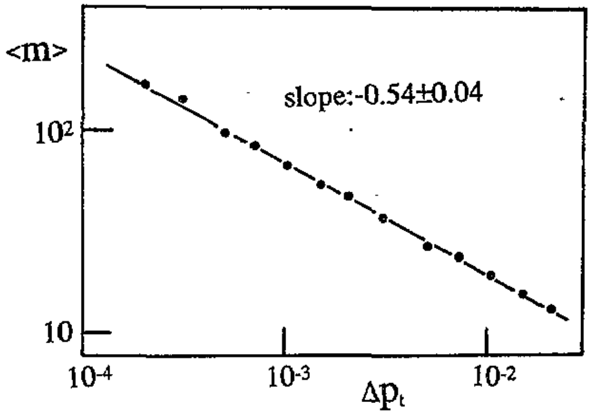

Figure 2. The log–log plot of the typical lifetime $\langle m \rangle$ of start–stop waves against $\Delta p_t (= 1 - p_t)$. The typical lifetime scales as $\langle m \rangle \approx \Delta p_t^{-0.54} \pm 0.04$.

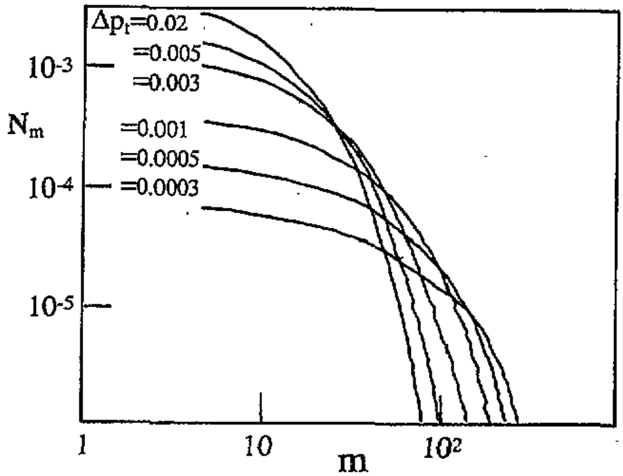

Figure 3. The log–log plot of the cumulative lifetime distribution N_m against lifetime m for $\Delta p_t = 0.02$, 0.005, 0.003, 0.001, 0.0005 and 0.0003.

We study the scaling behaviour of the lifetimes in the steady state near $p_t = 1$ after a sufficient number of time steps. We define the typical lifetime $\langle m \rangle$ of start–stop waves as

$$\langle m \rangle \equiv \sum_{m=1}^{\infty} m^2 n_m \Big/ \sum_{m=1}^{\infty} m n_m \tag{5}$$

where m is the lifetime of the start–stop waves and n_m is the lifetime distribution (the density of start–stop waves with lifetime m per one time step). Figure 2 shows the log-log plot of the typical lifetime $\langle m \rangle$ against $\Delta p_t (= 1 - p_t)$. The typical lifetime $\langle m \rangle$ scales as

$$\langle m \rangle \approx \Delta p_t^{-\nu} \qquad \text{with } \nu = 0.54 \pm 0.04. \tag{6}$$

The scaling behaviour of the lifetime depends little on the system size L under the condition $\Delta p_t \ll 1$ and sufficiently large L. The cumulative lifetime distribution N_m is defined as

$$N_m \equiv \sum_{m'=m}^{\infty} n_{m'}. \tag{7}$$

Figure 3 shows the log–log plot of the cumulative lifetime distribution N_m against lifetime m for $\Delta p_t = 0.02, 0.005, 0.003, 0.001, 0.0005, 0.0003$. We plot the rescaled cumulative lifetime distribution against the rescaled lifetime. Figure 4 shows the log–log plot of the rescaled cumulative lifetime distribution $N_m \Delta p_t^{-1.1}$ against the rescaled lifetime $m\Delta p_t^{0.54}$ for the data in figure 3. All data collapse onto a single curve. The cumulative lifetime distribution satisfies the scaling form

$$N_m(\Delta p_t) \approx \Delta p_t^{1.1} f(m\Delta p_t^{0.54}). \tag{8}$$

Therefore, the lifetime distribution n_m is described in terms of the scaling form

$$N_m(\Delta p_t) \approx \Delta p_t^{1.64} f'(m\Delta p_t^{0.54}). \tag{9}$$

We study the scaling behaviour of the interval of start–stop waves in the steady state after a sufficient number of time steps. Its interval is the distance between a start–stop wave and the next start–stop wave. We define the typical interval $\langle s \rangle$ of start–stop waves as

$$\langle s \rangle \equiv \sum_{s=1}^{\infty} s^2 n_s \bigg/ \sum_{s=1}^{\infty} s n_s \tag{10}$$

where n_s is the interval distribution (or the density of start–stop waves with interval s). Figure 5 shows the log–log plot of the typical interval $\langle s \rangle$ against Δp_t. The interval $\langle s \rangle$ scales as

$$\langle s \rangle \approx \Delta p_t^{-\nu'} \qquad \text{with } \nu' = 0.50 \pm 0.04. \tag{11}$$

The scaling exponent ν' of the interval agrees with the exponent ν of the lifetime within numerical accuracy. The cumulative interval distribution N_s is defined as

$$N_s \equiv \sum_{s'=s}^{\infty} n_{s'} \tag{12}$$

where n_s is the interval distribution. Figure 6 shows the log–log plot of the cumulative interval distribution N_s against interval s for $\Delta p_t = 0.02, 0.01, 0.005, 0.002, 0.001, 0.0005$. We plot the rescaled cumulative interval distribution against the rescaled interval. Figure 7 shows the log–log plot of the rescaled cumulative interval distribution $N_s \Delta p_t^{-0.50}$ against the rescaled interval $s\Delta p_t^{0.50}$ for the data in figure 6. All data collapse onto a single curve. The cumulative interval distribution satisfies the scaling form

$$N_s(\Delta p_t) \approx \Delta p_t^{0.50} g(s\Delta p_t^{0.50}). \tag{13}$$

Therefore, the interval distribution is described by the scaling form

$$n_s(\Delta p_t) \approx \Delta p_t^{1.0} g'(s\Delta p_t^{0.50}). \tag{14}$$

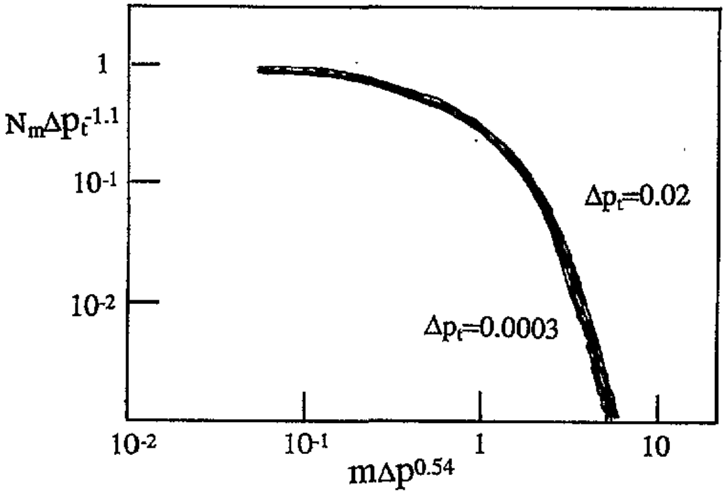

Figure 4. The log–log plot of the rescaled cumulative lifetime distribution $N_m \Delta p_t^{-1.1}$ against the rescaled lifetime $m \Delta p_t^{0.54}$ for the data in figure 3.

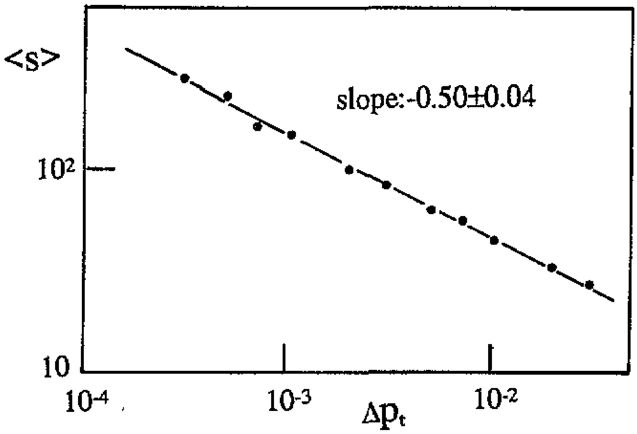

Figure 5. The log–log plot of the typical interval $\langle s \rangle$ of start–stop waves against Δp_t. The typical interval scales as $\langle s \rangle \approx \Delta p_t^{-0.50 \pm 0.04}$.

4. Discussion

We discuss our results with the previous results obtained from other models. Krug and Sphon [9] gave a mapping between the rough surface problem and CA model 184. The mapping corresponds to the deterministic case of $p_t = 1$. The random initial configuration with density 0.5 evolves to a particular configuration with the alternate spacing which corresponds to the flat surface. The configuration can be described as a pairwise ballistic annihilation process, the 'particles' and 'holes' ('traffic jams' and 'empty waves' in the traffic terminology used here) being pairs of 1's and 0's. In the process, pairs of particles and holes are not newly created and a random initial state is relaxed to the 'ordered' antiferromagnetic final configuration. In the present paper, pairs of particles and holes are created with a small probability Δp_t near $p_t = 1$. However, for $p_t < 1$, the pairwise

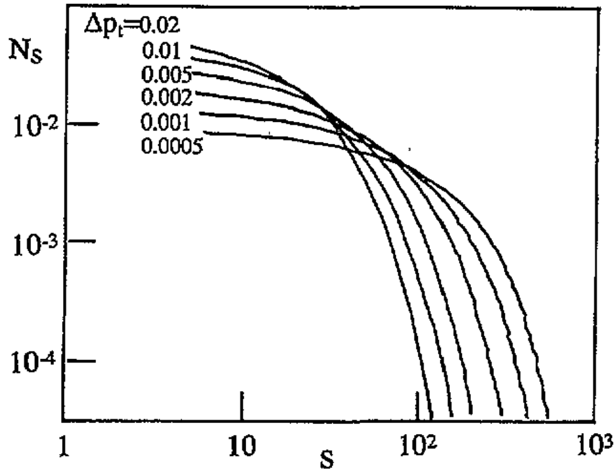

Figure 6. The log–log plot of the cumulative interval distribution N_s against the interval s for $\Delta p_t = 0.02$, 0.01, 0.005, 0.002, 0.001 and 0.0005.

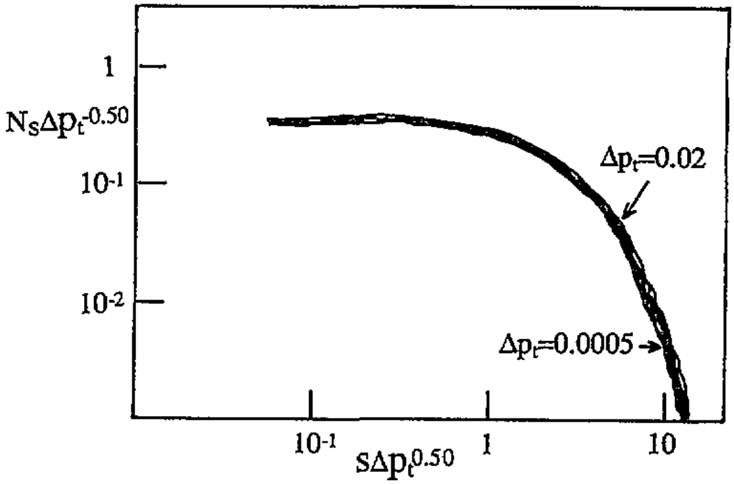

Figure 7. The log–log plot of the rescaled cumulative interval distribution $N_s \Delta p_t^{-0.50}$ against the rescaled interval $s \Delta p_t^{0.50}$ for the data in figure 6.

creation of particles and holes does not always occur. The resulting problem near $p_t = 1$—two-speed ballistic annihilation with pair creation—was studied in the context of 1D crystal growth where the two species of particles correspond to up and down steps on the surface and pair creation corresponds to nucleation of islands [28].

Following the analysis of Krug and Spohn [28], we derive the value of the scaling exponent ν. In the steady state, the particle and hole positions are uncorrelated and the total density in the steady state is obtained from a simple balance equation. Let ρ_+, ρ_- denote the densities of the two species. In the steady state, the creation rate Δp_t is balanced by the pairwise annihilation rate $2\rho_+\rho_-$ where the particles move with unit velocity. In the symmetric case $\rho_+ = \rho_-$ of interest here, this leads immediately to

$$\rho = \rho_+ + \rho_- = \sqrt{2\Delta p_t}. \tag{15}$$

Since the typical distance between traffic jams $\langle s\rangle \approx 1/\rho$, it follows that the exponent $\nu' = 1/2$ in equation (11) exactly. Moreover, it is clear from the ballistic nature of the process that lifetimes and distances scale in the same way, so that $\nu = \nu'$. Also, the exponent of the prefactor of the distribution (14) is simply set by the sum rule $\sum s n_s = L$ to be $2\nu = 1$.

We consider the boundary condition. The open boundaries used here drive the system to the 'critical' density $\frac{1}{2}$ for $p_t = 1$, as was first shown by Krug [29] for the fully-stochastic asymmetric simple exclusion process and therefore to ensure symmetry between 'traffic jams' and 'empty waves' ($\rho_+ = \rho_-$). The same behaviour would be observed in a system with periodic boundary conditions prepared at density 1/2.

We consider the limit of $\Delta p_t = 1/L$ in which a car stops temporarily over L sites per unit time. By replacing Δp_t with $1/L$ in equation (6), the typical lifetime $\langle m\rangle$ of traffic jams scales as

$$\langle m\rangle \approx L^{\nu} \qquad \text{with } \nu = 0.54. \tag{16}$$

By replacing Δp_t with $1/L$ in equation (8), the cumulative lifetime distribution $N_m(L)$ satisfies the finite-size scaling form

$$N_m(L) \approx L^{-\beta} f(mL^{-\nu}) \qquad \text{with } \beta = 1.1. \tag{17}$$

Similarly, by replacing Δp_t with $1/L$ in equations (11) and (13), the typical interval $\langle s\rangle$ and the cumulative interval distribution $N_s(L)$ scale respectively as

$$\langle s\rangle \approx L^{0.50} \tag{18}$$

$$N_s(L) \approx L^{0.50} g(sL^{-0.50}). \tag{19}$$

These results (16)–(19) are consistent with the results obtained from the previous model [25] in which a temporary stopping of a particle was added to the 1D fully-asymmetric exclusion model with open boundaries. The scaling relation $\beta = 2\nu$ was derived from a simple scaling argument [25].

In the cumulative lifetime distribution (8), the scaling function $f(x)$ is not power law even for $x \ll 1$. The scaling (8) is due to the creation and annihilation of traffic jams near $p_t = 1$. For $\Delta p_t \ll 1$, the lifetime distribution scales by Δp_t. However, the lifetime distribution in the CA model of Nagel and co-worker [22, 23] is presented by a power law. Traffic jams are not described by the ballistic annihilation process with creation. The scaling structure of our model is definitely different from that obtained by Nagel. Our model is very simple but the model of Nagel is complex. A comparison between our model and the model of Nagel is difficult since the definition of jam size in [22, 23] is very different from the one used here.

5. Summary

We have presented the 1D asymmetric exclusion model with stochastic transition probability p_t for freeway traffic. We found that near $p_t = 1$, the start–stop waves appear with various sizes and the system is driven asymptotically into a steady state exhibiting a self-organized criticality. We have investigated the scaling behaviours of the lifetime and interval in start–stop waves. We have shown that the typical lifetime $\langle m\rangle$ and typical interval $\langle s\rangle$ scale as $\langle m\rangle \approx \langle s\rangle \approx \Delta p_t^{-\nu}$ with $\nu = 0.5$. We found that the lifetime and interval distributions satisfy the scaling forms (9) and (14).

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